Mathematical probability: student's misconception in higher education

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Abstract. Probability is one of the mathematical concepts that are abstract for students. Probability concepts used in daily life. As a basis of statistics, students used the probability concept. This topic is one of the compulsory subjects learned in the Mathematics Education Study Program of Universitas Ahmad Dahlan. However, there are still many students who have difficulties in understanding the concept of probability. This study aims to describe students' misconceptions in solving Probability problems. The research used descriptive method. The subjects in this study were the fourth-semester students of Mathematics Education Study Program Class C of Universitas Ahmad Dahlan. The results of this study found that there are four misconceptions made by students in solving Probability problems. First, students' error in interpreting questions. Second, students' errors in the procedure of proving the theorems of probabilities. Third, students' misconceptions in the application of Bayes' rules while they are solving problems. Fourth, students' error in calculating the possibility of an event.

1. Introduction

Mathematics is the science that underlies all types of sciences. Mathematics has a substantial contribution to building human thought patterns, especially building reasoning in problem-solving [1]. In daily activities, humans and mathematics cannot be separated [2,3,4]. Mathematics has also become a problem-solving tool in other fields like science [5]. Mathematics has several characteristics that are explicitly categorized. Probability, or commonly known as Probability Theory, is included in the category of applied mathematics [6]. Probability is a concept that is difficult to learn and abstract [7]. In understanding the Probability concept, the most important thing needed is understanding the basic statistics. In mathematical learning, it shows that procedural knowledge supports conceptual knowledge and vice versa [8]. Therefore, understanding this concept becomes an essential tool to helps students to understand probabilities [9]. The primary material in learning probabilities includes the sample space, population, event, probability of an event, relationship between events calculation techniques, random variables, Bayes' rules, conditional event, and others.

Meanwhile, students still often make mistakes in solving probability problems. The error analysis method used to understand errors that are often made by students [10,11]. Misunderstanding of students in solving probability problems implies a lack of understanding of basic scientific concepts and facts [12]. Lack of understanding experienced by students is one of the causes of student errors in solving probability problems. Some students' mistakes made because of the lack of ability to read the questions command, the application of Bayes' rules, and combined calculation [1,12,13,14].

It was found that the result of midsemester examination score of Probability Theory of 4th-semester Mathematics Education students of Universitas Ahmad Dahlan class C was classified low. Based on the student answer sheet, some errors were made by students while solving Probability problems. Therefore, the researchers want to analyze the errors of mathematics education students in solving Probability problems. The researchers also want to evaluate the learning process in the Probability course. The research focuses on the errors made by students. Students' errors because of the lack of understanding of the concepts of probability calculation techniques and proving procedure of theorems of the relationship between the probability of events. This study aims to find and describe the errors made by students in solving probability problems.

2. Method

This research used descriptive qualitative method. In this study, researchers highlighted several errors made by students in solving Probability problems [15]. The subject of this research was 4th-semester students of Mathematics Education Universitas Ahmad Dahlan class C, who consists of 29 students. The subject of this research consisted of 23 female students and six male students. There were three stages used in analyzing students' errors in Probability problems, namely preparation, implementation, and data analysis. In the preparation stage, the researcher prepares a test instrument consisting of 4 questions that must be completed by the student. The second stage is implementation; researchers asked students to complete the test independently. In the third stage, the researchers analyzed the results of research in the form of students' answer sheets.

The data is analyzed and described inductively. Researchers described through stages of data collection, data reduction, data display, and drawing conclusion [16, 17]. The researchers used triangulation of data source to ensure validity of the data [17].

3. Result and Discussion

Based on data taken in the analysis stage, it was found that there are four common errors made by students in solving probability problems. There are students' errors in interpreting the problem, students' errors in proving the properties of probability, students' errors in applying Bayes' rules, and students' errors in calculating the probability of an event. All errors found, along with the number of students who did, are presented in Table 1.

Table 1. Type of students' errors

Code	Type of errors	Frequency
A	Misinterpreting the problem	27
B	Proving procedural error	25
C	Low understanding the application of Bayes' rules	6
D	Inaccurate in calculating the probability of an event	14

The first error is interpreting the problem. This error causes students to make mistakes in using the combination and permutations rules. The researcher names this error by code A. In mistake A, there are two types of errors and some possible reasons that make students making this mistake. The description of error A presented in Table 2.

Table 2. Mistake A

Types of students' errors	Possible Reasons
Misinterpreting the problem	Students' misunderstanding the use of combination and permutation rules in solving a problem.
	Inaccurate in calculation procedure using a combination.

Figure 1 shows that students wrong in solving problems use the combination problem. The mistake made by students is wrong in interpreting the question. Students consider that out of the ten available people will be formed into five groups, with each group having two members. So, the students calculate using C_2^{10}. Means that from 10 peoples will be taken two people for each group. In this problem, students should do calculations using a combination rule for each group. Combination for the first group until the fifth group in a row are $C_1^1, C_1^3, C_1^5, C_1^7, C_1^9$. Then multiply the results of these combinations.
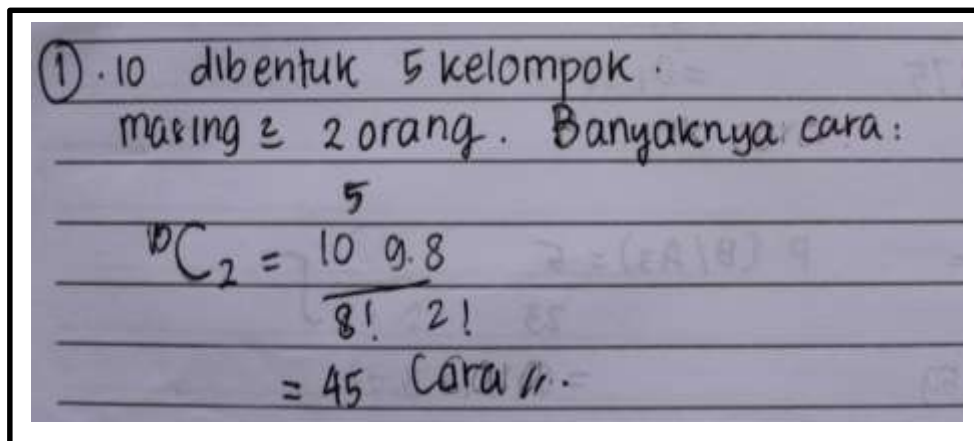

Figure 1. Error in using combinations rule

The second error is the students' errors in the procedure of proving the theorems of probability. The second error is given Code B. In error B; there are one type of student errors and three possible reasons that cause students to make these errors. The description of mistake B is shown in Table 3.

Table 3. Mistake B

Types of students' errors	Possible Reasons
Proving procedural error	Forget the basic theorems of probability
	Wrong in defining an event
	Students' misunderstanding the flow of proof of a theorem.

Figure 2 shows that students were wrong in carrying out the procedure of proving a theorem. Problem number two can be solved by defining events related to the theorem, namely defining events $A, B, A \cap B, A \cup B$. Then substitute the probability of event $A \cap B$ to the equation of probability of event $A \cup B$. It was found that students cannot define the probability of an event, namely $S = P(A) + P(B) + P(A \cap B)$. Students consider that event A is area A which is not shaded (see in Figure 2), it also applies to event B. This error makes students making mistakes in proving procedures. Accordance with the statement of the respondent that they do not understand the flow of proving a theorem.
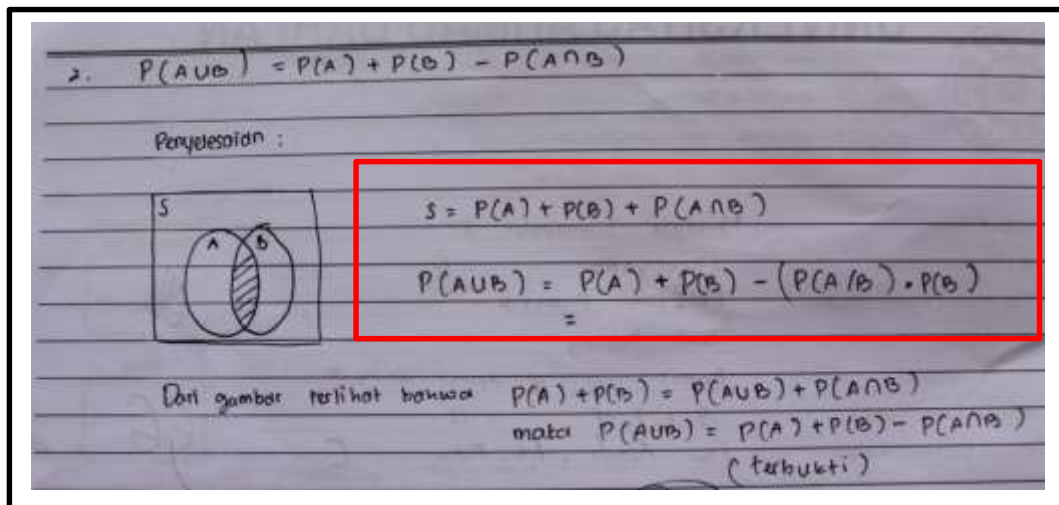
Figure 2. Students' errors in defines an event

The third students' error is applying Bayes' rules in solving the problem. This third error is given Code C. There are one type of mistake and two possible reasons that cause this mistake occurs. The type of error and their reasons can be seen in Table 4.

Table 4. Mistake C

Types of students' errors	Possible Reasons
Low understanding the application of Bayes' rules	Carelessly in calculating conditional probabilities
	Lack of concentration

Figure 3 shows the students' error in determining the conditional probabilities. Students should have calculated $P(B|A_1) = \frac{10}{30}$. This type of students' error caused by they were considered that $P(B|A_1)$ is the probability of event B but with the universe of all students, not students in the event A_1. This error also occurs in $P(B|A_2)$ and $P(B|A_3)$.
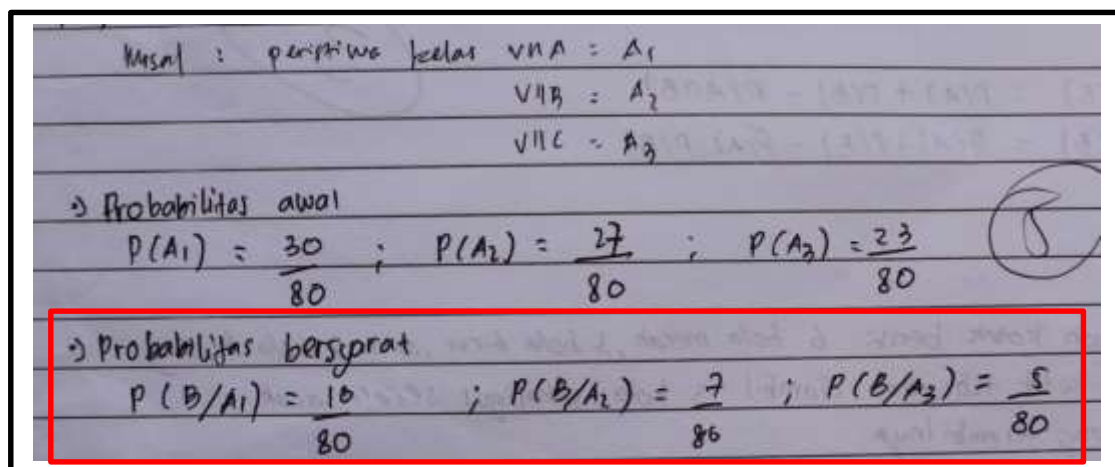

Figure 3. Students' errors in determine the conditional probability

The fourth mistake is that students are weak in understanding the concept of calculating the probability of an event. The fourth error is given by code D. There are two types of mistakes, and each of the possible reasons that cause this error occurs. The types of errors and their reasons can be seen in Table 5.

Table 5. Mistake D

Types of students' errors	Possible Reasons
Lack of understanding the concept of probability	Students' misunderstanding the concept of probability
	Incorrect in interpreting problems.
	Don't know the probability formula for an event
Inaccurate in calculating the probability of an event	Carelessly substitute the probability value of an event.

Figure 4 shows the students' error in understanding the concept of the value of a probability. Students find that the probability of an event is more than one. This is contrary to the concept of probability value is $0 \le P(A) \le 1$.

Figure 4. Do not understand the concept of the value of the probability

Figure 5 shows the student's mistake, namely misinterpreting the problems. In the problem, it is said that the ball is taken at once. Besides, the student considers that taking the ball is done one by one without return. So, the student is wrong in calculating the probability value of an event.
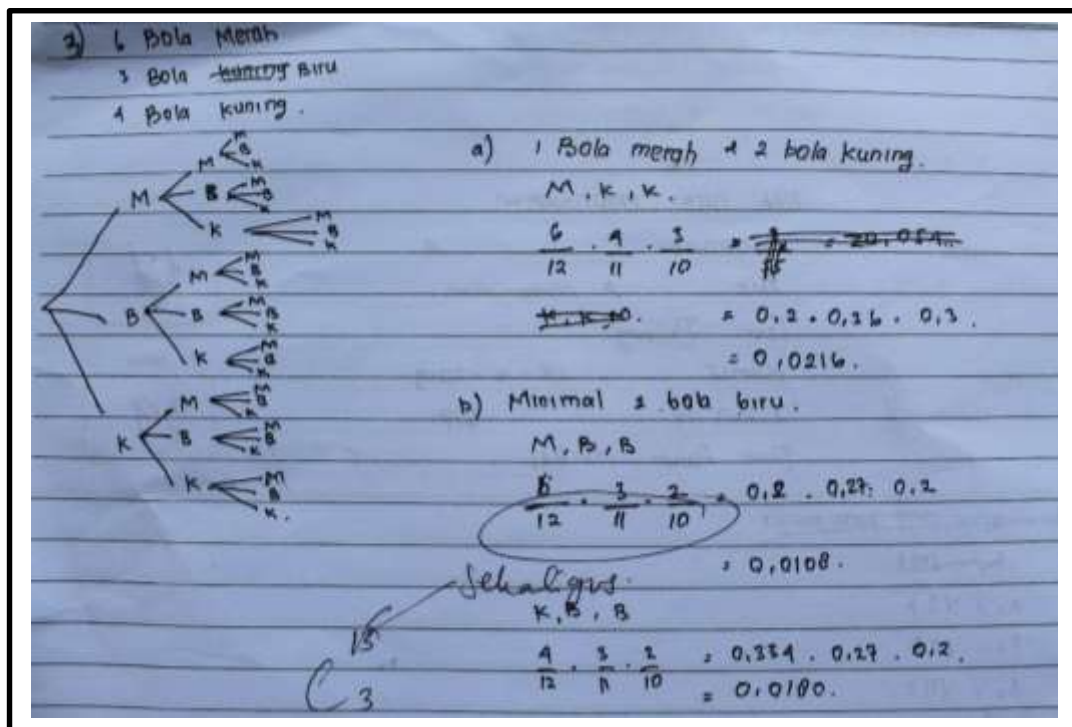
Figure 5. Students' error in interpreting problem

Error calculation of the probability is also done by the student because of the lack of understanding of the student in the calculation of the probability of an event. This is shown in Figure 6, that the student doesn't know the formula for calculating the probability of an event. Students should write the formula for the probability of an event, namely $P(A) = \frac{n(A)}{n(S)}$. However, the student only writes only $n(A)$.
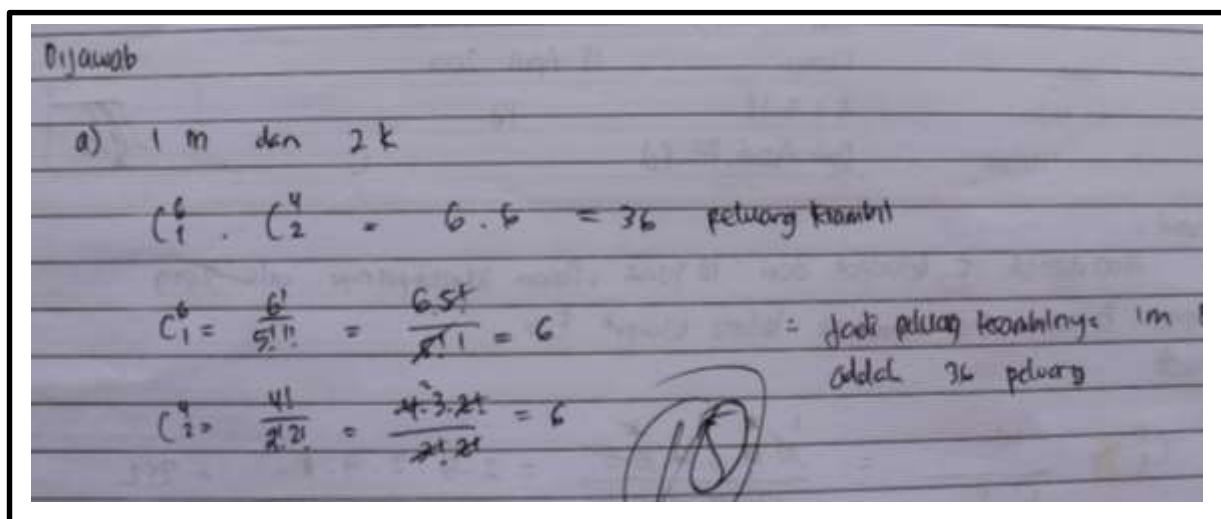

Figure 6. Incorrect in determining probability value

Based on the interview, there are several reasons that cause students' errors in solving probability problems. There are students were having trouble focusing in learning process, do not understand the basic concepts of probability and carelessly in calculating the probability of an event. They were also giving suggestions about the learning method which is desired by students, namely through discussion method, because students will feel more understanding when the teacher delivers the material directly.

This research also supports the result of Rahayuningrum and Setyawan' study that students' error occur when they were misunderstanding the purpose of the questions, misunderstanding the concepts, and completing the problem steps [18]. It also supports Triliana and Asih study related the analysis of students' error in solving probability problem. This research supports the evidence that students made errors in solving and interpreting the probability problems [19]. Also, students' error in calculating the probability of an event, specifically in Bayes' rule. It also found that this research is not only analyzing students' error but also students' suggestion in learning method used while studying probability. For the further study, the researchers need to analyses that discussion method that used to minimalized the students' misconception in learning probability.

4. Conclusion

In this study, four concept errors were obtained by students in solving probability problems, including errors in interpreting questions. Second, mistake in the procedure of proving the theorems of probabilities. Third, students do not understand the application of Bayes rules in problem-solving. Fourth, students are still lacking in the concept of calculating the probability of an event. The mistake most made by students are wrong in interpreting the problem, causing several other errors. Evaluation of this problem is the teacher should use in the learning method that needed by the student so as they minimize the mistakes made by the student.

Acknowledgment

References

[1]   Phonapichat P, Wongwanich S and Sujiva S 2014 *An Analysis Of Elementary School Students' Difficulties In Mathematical Problem Solving. Procedia-Social And Behavioral Sciences* **116** 3169 – 74

[2]   Triliana T and Asih E C M 2019 Analysis of students' errors in solving probability based on Newman's error analysis *J. Phys. Conf. Ser.* **1211** 012061

[3]   Chotimah S, Bernard M, and Wulandari S M 2018 Contextual approach using VBA learning media to improve students' mathematical displacement and disposition ability *In Journal of Physics: Conference Series* **948** 012025

[4]   Leibovich T, Katzin N, Harel M, and Henik A 2017 From sense of number to sense of magnitude: The role of continuous magnitudes in numerical cognition *Behavioral and Brain Sciences*

[5]   Retnawati H, Arlinwibowo J, Wulandari N F, and Pradani R G 2018 Teachers'difficulties And Strategies In Physics Teaching And Learning That Applying Mathematics *Journal Of Baltic Science Education* **17** 120 – 35

[6]   Von Plato J 1994 *Creating Modern Probability: Its Mathematics, Physics And Philosophy in Historical Perspective* (UK: Cambridge University Press)

[7]   Konold C 1991 *Understanding Students' Beliefs About Probability in Radical Constructivism in Mathematics Education* (in Radical Constructivism in Mathematics Education) ed Von Glasersfeld E (Springer Vol 7) 139 - 56

[8]   Rittle-Johnson B, Schneider M, and Star J R 2015 Not a one-way street: Bidirectional relations between procedural and conceptual knowledge of mathematics *Educational Psychology Review* **27** 587 – 97

[9]   ÇEPNİ S, ÜLGE B B, and ORMANCI 2017 Pre-Service Science Teachers' Views Towards The Process Of Associating Science Concepts With Everyday Life *Journal Of Turkish Science Education (TUSED)* **14** 1 – 15

[10]   Lai C F 2012 *Error analysis in mathematics* (Oregon: Behavioral Research and Teaching University of Oregon)

[11]   Abdullah A H, Abidin N L Z, and Ali M 2015 Analysis of students' errors in solving Higher Order Thinking Skills (HOTS) problems for the topic of fraction *Asian Social Science* **11** 133 – 42

[12]   Mestre J 2001 Cognitive Aspects Of Learning and Teaching Sciences *From: Chapter 3 Of Teacher Enhancement For Elemetary And Secondary Science And Mathematics: Status, Issues, And Problems* Eds S.J. Fitzsimmons & L.C. Kerplelman, (USA: Washington, D.C) 80

[13]   Budgett S and Pfannkuch M 2019 Visualizing chance: tackling conditional probability misconceptions *Topics and Trends in Current Statistics Education Research* (Springer) 3-25

[14]   Ang L H and Shahrill M 2014 Identifying students' specific misconceptions in learning probability *International Journal of Probability and Statistics* **3** 23-29

[15]   Rokhimah S, Suyitno A, and Sukestiyarno Y L 2015 Students Error Analysis In Solving Math Word Problems Of Social Arithmetic Material For 7th Grade Based on Newman Procedure *Proceeding in International Conference On Conservation For Better Life* (Semarang: Unnes) p. 349 – 56

[16]   Miles M B & Huberman M A 2012 *Analisis Data Kualitatif: Buku Sumber Tentang Metode-Metode Baru. Universitas Indonesia_UI Press (11th ed.)* (Jakarta: Universitas Indonesia)

[17]   Moleong L J 2017 *Metodologi Penelitian Kualitatif (Edisi Revisi)* (PT. Remaja Rosda Karya)

[18]   Rahayuningrum A & Setyawan F 2019 Analisis Kesalahan Siswa Kelas V SD dalam Menyelesaikan Masalah Sehari-hari yang Berkaitan dengan Operasi Hitung Pecahan *Seminar Nasional Pendidikan Matematika Ahmad Dahlan*

[19]   Triliana T & Asih E C M 2019 Analysis of students' errors in solving probability based on Newman's error analysis *Journal of Physics: Conference Series* **1211** 012061 (IOP Publishing)